For all problems, draw all sketches, diagrams, and Force Body Diagrams (FBDs) that are needed. In some cases, you may need to add to provided diagrams. You will likely get potential points for showing diagrams or providing correctly drawn vectors on FBDs for example.

To receive the maximum number of points it is best to follow the mnemonic "E-CAN-SU". ECANSU stands for Equations, Canceling/zeroing, $\underline{A} / g e b r a, \underline{N} u m b e r s, \underline{S}$ ignificant figures, and Units. You can remember this as Everyone CAN be Successful and Useful. Therefore, the first thing you do when solving a problem is write out the full starting equation you have been taught for a subject, then you cancel our or draw a line through the terms in the equations that go to zero, then you do any needed algebra to solve for the desired variable, then plug in the numbers, then report the answer to the correct number of significant figures, with the appropriate units. That is it.

You will earn points for writing the basic full equations with all terms. It is important for you to show me explicitly what terms goes to zero, if it is not shown, that simply means you forgot the term, compared to if a term is shown with a line through, it tells me you know it is zero and it was not simply forgotten. No points are awarded if numbers are plugged in first before the algebra. You can potentially earn points for using the correct sig figs or the requested sig figs and the proper units. A number without a unit is meaningless. Always remember: SHOW YOUR WORK and SHOW ME WHAT YOU KNOW!

## How to start and progress through a problem (see examples that follow)?

1) First read the problem. If appropriate, which in most cases it is, draw and label a sketch that describes the situation or problem. If a sketch is already provided, you may need to add to it, showing any missing or additional information. At first, the situation might not be clear after reading the problem one time. Reread the problem several times if necessary. If the situation is still not clear, try moving onto step (2) and come back to sketching a diagram when the situation/problem becomes more clear.
2) Write out what quantity or physical property the problem is asking you to solve for (use the correct terminology and symbols from the question). What are the units on this quantity, this will be important later. If this quantity is a vector, it should be appropriately drawn and labeled on the diagram made in step (1). The blue text in the following provided example is excessive for sure, and can be excluded all together, but perfectly okay if it helps you work through a problem. The level of blue text is not expected. See the real examples of a student homework solutions in Blackboard for comparison. Look carefully at where point values are assigned so you understand what is important. Basically, the points assigned follow "ECANSU". If you cannot see why points are awarded following ECANSU, come see me.
3) Write out the known and unknown quantities in the problem, in symbols, words, and/or numbers with the correct units. At a minimum, these should also appear on your diagram, and if applicable, with appropriate vector notation and direction.
4) Also, write down the important physics concepts that might be applicable to the problem. Where is the origin? What is your sign convention for example? What are key features or statements in the problem? Write down and explain what they are and why they are important.
5) Write down the full basic equation(s) you will need to solve for the desired quantity just using the symbols (wait to plug in the numbers later); these equations contain both your knowns and unknowns. Avoid writing down irrelevant equations; these receive no partial credit and simply clutter up the page.
6) Solve the above equation(s) and/or manipulate/combine them algebraically to arrive at a final equation that just has symbols in it. The quantity that you are interested in should be on the left side of the equation everything else should be on the right side of the equals sign in the equation. Check the units on this algebraic expression...will the units work correctly for the desired quantity listed in step (2)? You should check this before plugging in numbers. If you can make it to this point correctly in a problem you will most likely receive nearly full credit. A page of just numbers with no units or equations will most likely result in very little to no partial credit for problems, even if the answer is correct and boxed.
7) Finally, plug numbers into the final equations found in step (6) to get the numerical results. Ask yourself does this result seem reasonable? Is the number too large or too small? Is the sign (+ or -) correct and what you expected according to your diagram and sign convention? If not, go back and double check your work.
8) Display the final numerical answer rounded to the correct significant figures with the correct units all of which needs to be enclosed with a hand drawn box. If you cannot solve for the desired variable or quantity, list possible reasons why you think this is the case.

## Example Problem

As an entry level Criminal Investigator for the West Virginia State Police Department, you rely heavily on your skills obtained in your introductory physics courses from Marshall University. Today, your Captain assigns you to a recent accident, where a car, originally going around a sharp curve marked at $56.0 \mathrm{~km} / \mathrm{h}$ (kilometers per hour), has driven off the edge of a nearby cliff that is 58.0 m high. When you arrive on scene, you can easily see the point of the fiery impact was 0.136 km from the base of the cliff. (a) Did the car violate the $56.0 \mathrm{~km} / \mathrm{h}$ posted speed limit before plummeting off the edge of the cliff? (b) What was the final impact speed of the car in MPH (miles per hour)? (c) At what angle did the car impact the rocks below the cliff?

- We need to find (a) the horizontal component of velocity prior to the vehicle going off the cliff (the magnitude of the $x$-component of velocity at the top of the cliff), (b) the overall magnitude of the velocity of the car before impact/square root of the sum of the squares of the $x$ and $y$ components of velocity before impact) and (c) direction of the car before impact /inverse tangent of the ratio of the $y$ components of velocity over the $x$ components of velocity).
- We know that once the car leaves the cliff the only forces acting on the car, if we ignore air resistance, is the force due to gravity. This force due to gravity is due to an object having mass and being acted on by the Earth's a constant gravitational acceleration of $9.81 \mathrm{~m} / \mathrm{s}^{2}$ straight down in the $y$-direction. Once the car leaves the cliff, there are no forces, hence accelerations, in the $x$-direction (ignoring air resistance). Thus, the speed in the $y$-direction will increase from zero as the car plummets, but the speed in the $x$ direction will remain constant until impact. This is a classic projectile motion problem: find the time of flight
in one dimension, plug it into kinematic equations in the other dimension to get desired quantities, such as speeds and distances. Though it can be set anywhere, let's set the origin at
the top of the cliff and set the downward direction as negative. Note, drawing not to scale.
(a)
$y_{o}=0$
$y=-58.0 m$
$x_{o}=0$
$x=136 m$
$a_{x}=0$
+1) $a_{y}=-g \& g=9.81 \mathrm{~m} / \mathrm{s}^{2}$
(+1) $x=x_{o}+v_{o x} t+\frac{1}{2} a_{x} t^{2} \Rightarrow\left(x-x_{o}\right)=v_{o x} t$
+1 $y=y_{o}+y_{\text {oo }} t+\frac{1}{2} a_{y} t^{2} \Rightarrow\left(y-y_{o}\right)=-\frac{1}{2} g t^{2} \Rightarrow t=\sqrt{\frac{-2\left(y-y_{o}\right)}{g}}$
$y=-58.0 \mathrm{~m}$
$t=\sqrt{\frac{-2(-58.0 \mathrm{~m}-0)}{9.81 \mathrm{~m} / \mathrm{s}^{2}}}=\sqrt{11.8247 \mathrm{~s}^{2}}=3.4387 \mathrm{~s}$

$\left(x-x_{o}\right)=v_{o x} t \Rightarrow v_{o x}^{+1}=\frac{\left(x-x_{o}\right)}{t}=\frac{136 \mathrm{~m}}{3.4387 \mathrm{~s}}=39.5498 \frac{\mathrm{~m}}{\mathrm{~s}}$
Convert this $x$-component speed to $\mathrm{lm} / \mathrm{h}$ and report with 3 sig figs.
$v_{o x}=39.5498 \frac{\mathrm{~m}}{\mathrm{~s}} \times \frac{608}{1 \mathrm{~m} / \mathrm{m}} \times \frac{60 \mathrm{nin}}{1 \mathrm{hour}} \times \frac{1 \mathrm{~km}}{1000 \mathrm{~m}} \Rightarrow v_{o x}=142.379 \mathrm{~km} / \mathrm{h} \quad v_{o x}=v_{x}=142 \mathrm{~km} / \mathrm{h}$
This speed greatly exceeds the posted speed limit of $56 \mathrm{~km} / \mathrm{h}$, which is the likely cause of the accident.

$$
\begin{array}{ll}
v_{o x}=142.379 \mathrm{~km} / \mathrm{h} \times \frac{1000 \mathrm{~m}}{1 \mathrm{kmi}} \times \frac{1 \mathrm{mile}}{1609 \mathrm{~m}}=88.4891 \frac{\mathrm{miles}}{\text { hour }} & v_{o x}=v_{x}=88.5 \mathrm{MPH} \\
v_{\text {posted }}=56 \mathrm{~km} / \mathrm{h} \times \frac{1000 \mathrm{~m}}{1 \mathrm{~km}} \times \frac{1 \mathrm{mile}}{1609 \mathrm{~m}}=34.8042 \frac{\text { miles }}{\text { hour }} & v_{\text {posted }}=35 M P H
\end{array}
$$

These results seem reasonable.
(b) We have the $x$-component of velocity, 88.5 MPH, but we also need the $y$-component of the final velocity. The $y$-component of the velocity at any height would be the same as if the car was simply dropped off the cliff. This $y$-component velocity is independent of the $x$-component speed and $x$-position since there is only acceleration in the $y$-direction. The initial $y$-component of velocity is zero.
+1)
(+1) $v_{y}=v / v_{y}+a_{y} t \Rightarrow v_{y}=-g t=-9.81 \mathrm{~m} / \mathrm{s}^{2} \times(3.4387 \mathrm{~s})=-33.7336 \mathrm{~m} / \mathrm{s}$
$+1$
$+1$
$+1$

$$
v_{y}=-33.7336 \text { mf s } \times \frac{1 \text { mile }}{1609 \mathrm{hn}} \times \frac{3600 \mathrm{~s}}{1 \text { hour }}=-75.476 \frac{\text { miles }}{\text { hour }} \quad v_{y}=-75.5 \mathrm{MPH}
$$

Note the negative sign indicates that the velocity is in a direction down past the origin.
To find the magnitude of the impact velocity we take the square root of the sum of the squares of the $x$ and $y$ components of velocity before impact.
+1) $v_{\text {impact }}=\sqrt{v_{x}^{2}+v_{y}^{2}}=\sqrt{(88.4891 M P H)^{2}+(-75.476 M P H)^{2}}=116.305 M P H$

$$
v_{\text {impact }}=116 M P H
$$

(c) To find the angle of impact we take the inverse tangent of the ratio of the y components of velocity over the $x$ components of velocity. $\theta=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)=\tan ^{-1}\left(\frac{-75.476 M P H}{88.4891 M P H}\right)=-40.4622^{\circ} \quad \theta_{\text {impact }}^{+1+2}=-10.5^{\circ}$
Note the negative sign indicates that the angle is below the horizontal for ow as measured from the positive $x$-axis).

## Significant Figures Review

## Rules for Determining the Number of Significant Figures:

1. All non-zero digits are significant.
2. All zeros between non-zero digits are significant.
3. All beginning zeros are not significant.
4. Ending zeros are significant if the decimal point is actually written in but not significant if the decimal point is an understood decimal (the decimal point is not written in).

Examples of the Significant Figure Rules:

1. All non-zero digits are significant.

| 652 | has 3 significant figures |
| :--- | :--- |
| 25.698 | has 5 significant figures |
| 19.8625897 | has 9 significant figures |
| 12.56 | has 4 significant figures |
| 9.2 | has 2 significant figures |

2. All zeros between non-zero digits are significant.

8,006 has 4 significant figures
12.5005 has 6 significant figures

308 has 3 significant figures
1.01 has 3 significant figures
90.02 has 4 significant figures

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3. All beginning zeros are not significant.

| 0.0000013 | has 2 significant figures |
| :--- | :--- |
| 0.05 | has 1 significant figure |
| 0.002004 | has 4 significant figures |
| 0.1 | has 1 significant figure |
| 0.000504 | has 3 significant figures |

4. Ending zeros are significant if the decimal point is actually written in, but not significant if the decimal point is an understood decimal.

| 69.500 | has 5 significant figures |
| :--- | :--- |
| 44.000000 | has 8 significant figures |
| 100. | has 3 significant figures |
| 100 | has 1 significant figure |
| 604,000 | has 3 significant figures |
| 1,050 | has 3 significant figures |

5. Scientific Notation rules for logs: If taking the log of a number with N significant figures, the result should have N decimal places. The number in front of the decimal place is not a significant figure.*
6. Scientific Notation rules for exponents in powers of 10: When raising 10 to a power, if the power of 10 has N decimal places, the result should have N significant figures. *
*https://wpsites.ucalgary.ca/chem-textbook/significant-figures-in-logarithms/

## Review of Significant Figures in Calculations

Addition and Subtraction - The answer to an addition or subtraction operation must not have any digits further to the right than the 'shortest' number to be added. In other words, the answer should have as many decimal places as the number to be added that has the smallest number of decimal places (or the least precise number to be added).

Example 1:

$$
15.6987 \mathrm{~cm}+5.327 \mathrm{~cm}+6.23 \mathrm{~cm}=27.2557 \mathrm{~cm}=27.26 \mathrm{~cm}
$$

## Example 2:

$$
19 \mathrm{~m}+0.00098 \mathrm{~m}=19.00098 \mathrm{~m}=19 \mathrm{~m}
$$

In this case, the number 19 has no digits beyond the decimal. Therefore, all numbers past the decimal point must be rounded off in the final answer. Thus, the answer is still 19.

## Example 3:

$$
68.9946 \mathrm{~cm}+9.67 \mathrm{~cm}+87.0 \mathrm{~cm}=165.6646=165.7 \mathrm{~cm}
$$

## Multiplication and Division

The answer for a multiplication or division operation must have the same number of significant figures as the number with the least number of significant figures.

## Example 4:

$$
(2.283 \mathrm{~cm}) \times(3.2 \mathrm{~cm})=7.3056 \mathrm{~cm}^{2}=7.3 \mathrm{~cm}^{2}
$$

The factor 2.283 cm has four significant figures, and the factor 3.2 cm has two significant figures. Therefore, the answer must have two significant figures. The mathematical answer of $7.3056 \mathrm{~cm}^{2}$ must be rounded to 7.3 $\mathrm{cm}^{2}$ in order for the answer to have two significant figures.

## Example 5:

$$
(25.0 \mathrm{~cm}) \times(4.00000 \mathrm{~cm})=100.000000 \mathrm{~cm}^{2}=100 . \mathrm{cm}^{2}
$$

The factor 25.0 cm has three significant figures, and the factor 4.00000 has six significant figures. The answer must be rounded to three significant figures. Therefore, the decimal must be written in to show that the two ending zeros are significant. If the decimal is omitted (left as an understood decimal), the two zeros will not be significant and the answer will be wrong.

## Example 6:

$$
(7.822 \mathrm{~cm}) \times(33 \mathrm{~cm})=258.126 \mathrm{~cm}^{2}=260 \mathrm{~cm}^{2}
$$

Here, the answer is rounded to two significant figures. We cannot have a decimal after the zero in 260 $\mathrm{cm}^{2}$ because that would indicate the zero is significant, the answer must have exactly two significant figures.

## Significant Figures with Mixed Operations

When doing a calculation that involves only multiplication and/or division, you can do the entire calculation then round the answer to the correct number of significant figures at the end. The same is true for a calculation that involves only addition and/or subtraction. But what about a calculation that involves mixed operations: both multiplication or division and addition or subtraction?

## Multiple Mathematical Operations

If a calculation involves a combination of mathematical operations, perform the calculation using more figures than will be significant to arrive at a value. Then, go back and look at the individual steps of the calculation and determine how many significant figures would carry through to the final result based on the above conventions.

## Example 7:

$$
X=((5.256+0.0016) / 34.6)-2.231 * 10^{-3}
$$

Calculate the value of $X$ using more digits than will be significant. In this case $X=0.149636052023$

Then, go back and look at each piece of the calculation to determine the significant figures. Using the mnemonic PEMDAS, Please Excuse My Dear Aunt Sally, for the correct order of operations Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction,
$5.256+0.0016=5.2576$ (the sum is limited to the thousandths place by 5.256 )
$5.2576 / 34.6=0.151954$ (the quotient is limited to 3 significant figures by 34.6)
$0.151954-0.002231=0.149723$ (the difference is limited to the thousandths place by 0.152 from the previous step, 0.152 would be the premature rounded results, don't round early).

The value initially obtained for $\mathrm{X}(0.14972275722543)$ should be rounded to have 3 significant digits
Therefore, the final answer is 0.150 or $1.50 \times 10^{-1}$.

Instead, if we were given 5.2 instead of 5.256 , our final answer would then have to be limited to 0.15 or $1.5 \times 10^{-1}$ or the tenth place by rules for addition.

Likewise, if we were given 34 instead of 34.6 with everything else the same, our final answer would then have to be limited to 0.15 or $1.5 \times 10^{-1}$ or two significant figures by rules for division.

## Example 8:

Another example, evaluate $0.5230 / 1.2941-(73.152-2.35) / 0.001$ * 5.72. Here 0.001 is meant to be thought of as an exact number, so it has only one significant figure. Everything to the left of the vertical line in the below calculations is not significant, but they are still included in calculations until the very end. The vertical line is like a place holder to indicate what numbers are significant and which are not. Numbers after the vertical line are not reported in the final answer, but are used in calculations. Rounding occurs only at the final step in a calculation. Rounding at every step is incorrect and will result in errors in your calculations.

```
\(0.5230 / 1.2941-(73.152-2.35) / 0.001 * 5.72=\)
\(0.4041 \mid 419-(70.80 \mid 2) / 0.001 * 5.72=\)
\(0.4041|419-7| 0802\) * \(5.72=\)
0.4041|419-4|04987.44 =
-4|04987.036
```

Keeping only the one trusted digit, the answer would be -400000 !

For a number in scientific notation, $\mathrm{Nx} 10^{\mathrm{x}}$, all digits in ' N ' ARE significant; the precision of " 10 " and " x " are NOT considered for significant figures in the final answer ( $x$ must be integer values). Example, $5.02 \times 10^{4}$ has THREE significant figures, "5.02". The precision of "10 and "4" are not considered when determining significant figures in the final answer.

For logs and exponents, we need to understand what is the mantissa, which is a fancy name for saying the numbers that follows to the right of the decimal point.

So, for exponents, when not using scientific notation, the number of sig figs is the same as the number of digits in the mantissa. For example, $10^{1.23}=16.9824$. The final answer should have 2 sig figs because there are 2 digits in the mantissa (.23). So, we report the answer in scientific notation with two sig figs, after rounding, $1.7 \times 10^{1}$.

For logs, it again depends on the numbers in the mantissa. For example, using a calculator will give the following:
$\log \left(\underline{2.73} \times 10^{-5}\right)=-4.563837353$
Here, the mantissa of the number to be logged is underlined, showing 3 significant figures ( 0.0000273 ). The same number of significant figures is underlined starting with the decimal point in the answer ('the characteristic' - a fancy way of saying the numbers to the left of the decimal point, are not include as part of the allowed sig figs). After rounding, the correct answer-4.564 is obtained.

Here is a nice short video for logs and exponents https://www.youtube.com/watch?v=mEwY4f4Tync

